Temperature Distribution in a Spinning Spherical Space Vehicle

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Solar heating of a thin-walled, spherical space vehicle, spinning in the interstellar space around an axis perpendicular to sun rays, is investigated. The interior of the vehicle is assumed to be highly evacuated, and the effect of internal radiation is included. The problem is reduced to solution of an ordinary differential equation that is linearized, and the subsequent analysis results in a series solution that may be used to obtain estimates of temperature fluctuations in the walls of the vehicle as a function of the rate of spin and of other pertinent parameters.

An important factor in the design of an artificial satellite or a space vehicle is the temperature distribution on its surface. This problem has received considerable attention before. An interesting special case is the temperature fluctuation in the skin due to spinning of the vehicle. For the case of cylindrical geometry, this has been recently investigated by Charnes and Raynor (1).[2]

In the present paper, the same problem is considered for the case of a spherical shell. Spherical geometry is a feature found frequently in recent satellite designs and deserves special attention. Internal radiation effects are also included because of their general importance in smoothing the temperature distribution in the skin of a satellite.

Statement of Problem

A formula for the temperature fluctuation in the skin of a spinning satellite when the spin axis is normal to the solar flux can be obtained from the solution of the governing partial differential equation, which for a shell of radius r and thickness h in spherical coordinates (r,θ,ψ) is simply

$$c\rho h \frac{\partial T}{\partial t} = \gamma \left(\frac{\partial^2 T}{\partial \theta^2} - \tan\theta \frac{\partial T}{\partial \theta} + \frac{1}{\cos^2\theta}\frac{\partial^2 T}{\partial \psi^2}\right) + Q \quad [1]$$

where

- T = absolute temperature
- t = time
- c = specific heat
- k = thermal conductivity
- ρ = density
- Q = heat sources due to external and internal radiation per unit area
- γ = kh/r^2, satellite skin conductance parameter

The coordinate system used is shown in Fig. 1, and the heat equation itself is of the type discussed, for example, by Eckert and Drake (2), namely

$$c\rho(\partial T/\partial t) = k\nabla^2 T + Q' \quad [2]$$

where the term Q' represents the sum of all the heat sources and sinks per unit volume and time. The term Q consists of three parts: direct solar heat, radiation emitted externally and internally, and internal radiation absorbed. In the present case the absorbed energy due to direct solar radiation Q_s is

$$Q_s = \bar{\alpha} S \cos^{+}(\psi - 2\pi f t) \cos\theta \quad [3]$$

where f = the rate of spin, and the symbol $\cos^{+}(\psi - 2\pi f t)$ means a half-wave rectified cosine function shown in Fig. 1.

Assuming a negligible temperature drop through the shell thickness h, each shell element will emit externally a heat quantity $\sigma\bar{\epsilon}T^4$ and internally $\sigma\epsilon_i T^4$; here, $\bar{\epsilon}$ is the overall emissivity of the exterior surface, and ϵ_i is the emissivity of the internal surface. Consequently, the absorbed internal radiation Q_i will contribute

$$Q_i = (\bar{\alpha}S/4)\beta \quad [4]$$

with β being the ratio of ϵ_i to $\bar{\epsilon}$. Eq. [4] results from the integration of the radiation contribution of each area element, with the proper angle factor, through the entire interior of the shell; it can be readily shown that Q_i is independent of both the rate of spin and the surface temperature distribution.

The differential Eq. [1] is nonlinear, and the method of obtaining an exact analytical solution is not known. Therefore, certain approximations will now be made. The term T^4 will be expanded with reference to the temperature T_∞ defined by

$$T_\infty = \left(\frac{\bar{\alpha}S}{\pi\sigma\bar{\epsilon}}\right)^{1/4} \left\{\frac{[\cos\theta + (\pi\beta/4)]}{(1+\beta)}\right\}^{1/4} \quad [5]$$

Eq. [5] may be obtained from Eq. [1] for $f \to \infty$ and conditions where the effects of conduction are negligible, i.e., when $\gamma \to 0$. Then

$$T^4 = (T + T_\infty - T_\infty)^4 \approx T_\infty^4[1 + (4/T_\infty)(T - T_\infty)]$$

and

$$T^4 \approx 4TT_\infty^3 - 3T_\infty^4$$

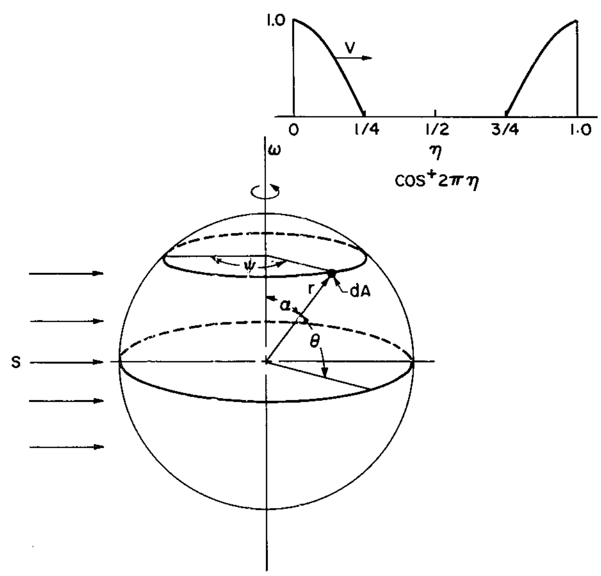

Fig. 1 Geometry of a spinning spherical shell

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[2] Numbers in parentheses indicate References at end of paper.

This kind of expansion is well justified for $|T - T_\infty| \ll T_\infty$, which condition will be realized for a sufficiently high rate of spin. Actually, the main objective of the present investigation is to find under what circumstances this condition applies; it will be shown below that the concept of the dimensionless thermal velocity ν_0 is helpful in that respect. High values of f are associated with high values of ν_0, and vice versa.

It may be shown that, for thin shells with $h \ll r$, Eq. [5] gives a realistic temperature distribution even for materials with a high thermal conductivity.

The present investigation is restricted to the temperature at $\theta = 0$, that is, at the "equator" of a spinning satellite. The equatorial region is of greatest interest, because this is the place where the spin-induced temperature fluctuations may be expected to be the largest. The temperature will then be given as deviation from the temperature T_∞, with $\theta = 0$. The term $\partial T/\partial\theta|_{\theta=0} = 0$ because of symmetry of the problem. An expression for $\partial^2 T/\partial\theta^2|_{\theta=0}$ may be obtained for a small γ and a sufficiently large f from Eq. [5] by differentiating twice and letting $\theta = 0$:

$$\left.\frac{\partial^2 T}{\partial\theta^2}\right|_{\theta=0} = -\frac{\bar{\alpha}S}{4\pi\sigma\bar{\epsilon}}\frac{T_\infty^{-3}}{1+\beta} = -\frac{1}{4}\frac{T_\infty}{1+(\pi\beta/4)}$$

It should be noted now that, since here $\partial^2T/\partial\theta^2$ represents the curvature of the temperature field in the θ direction, $\partial^2T/\partial\theta^2$ is always negative and has its minimum value when $\gamma \to 0$. The above reasoning is based again on the fact that, for a sufficiently large f, $|T - T_\infty|/T \ll 1$, and, therefore, $\partial^2T/\partial\theta^2$ assumes then essentially a constant value for a given θ and for any ψ.

Reduction to an Ordinary Differential Equation

A transformation of coordinates will be made to reduce the remaining terms of Eq. [1] to an ordinary differential equation.

In a spinning vehicle a "quasi-steady state" will be reached eventually; for a steady rate of spin, the temperature distribution will appear to be in the steady state as viewed by a stationary observer. From the point of view of the satellite, the heat source is traveling at a uniform speed, however. Therefore, the technique that applies to the moving heat sources for the quasi-steady state as discussed, for example, in Ref. 2, may be used here to advantage.

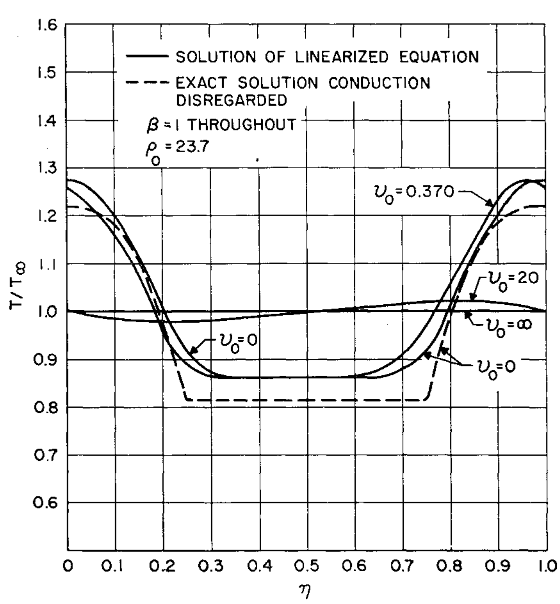


Fig. 2 Temperature distribution along the equator of a spinning spherical satellite for several dimensionless surface velocities ν_0

Letting $2\pi\eta = \psi - 2\pi ft$ is equivalent to saying that $T = T(\eta)$, $Q = Q(\eta)$. In the new coordinate system, one then has

$$\frac{\partial T}{\partial\psi} = \frac{1}{2\pi}\frac{\partial T}{\partial\eta}$$

$$\frac{\partial^2 T}{\partial\psi^2} = \frac{1}{4\pi^2}\frac{\partial^2 T}{\partial\eta^2}$$

Since, in the steady state, the total derivative of temperature with respect to time must be zero, in the case under consideration $dT/dt = 0$ for a stationary observer. Then

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + \frac{\partial T}{\partial\psi}\frac{\partial\psi}{\partial t} = 0$$

This expression is similar to the Stokes total time derivative used in hydrodynamics, discussed, for example, in Ref. 3. Therefore, making use of the previous definition of η, in the quasi-steady state

$$\partial T/\partial t = -f(\partial T/\partial\eta)$$

After all the substitutions have been made, and letting $k/c\rho = a$, the thermal diffusivity, for the situation at the equator of the satellite, Eq. [1] is as follows:

$$-\frac{f\gamma r^2}{a}\frac{dT}{d\eta} = \frac{\gamma}{4\pi^2}\frac{d^2T}{d\eta^2} - \frac{4\bar{\alpha}S[1+(\pi\beta/4)]}{\pi T_\infty}\left(T - \frac{3}{4}T_\infty\right) - \frac{\gamma T_\infty}{4[1+(\pi\beta/4)]} + \bar{\alpha}S\left(\cos^{+}2\pi\eta + \frac{\beta}{4}\right) \quad [6]$$

This may be rewritten by letting $T/T_\infty = \tau$ with T_∞ from Eq. [5]

$$\frac{d^2\tau}{d\eta^2} + b\frac{d\tau}{d\eta} - c\left(\tau + \epsilon - \frac{3}{4}\right) = -\frac{\pi c}{4}\frac{\cos^{+}2\pi\eta + (\beta/4)}{1+(\pi\beta/4)} \quad [7]$$

where

$$b = 4\pi^2 r^2 f/a$$

$$c = \frac{16\pi}{\gamma}\frac{\bar{\alpha}S}{T_\infty}\left(1 + \frac{\pi\beta}{4}\right)$$

$$\epsilon = \gamma\frac{\pi T_\infty}{16S\bar{\alpha}[1+(\pi\beta/4)]^2}$$

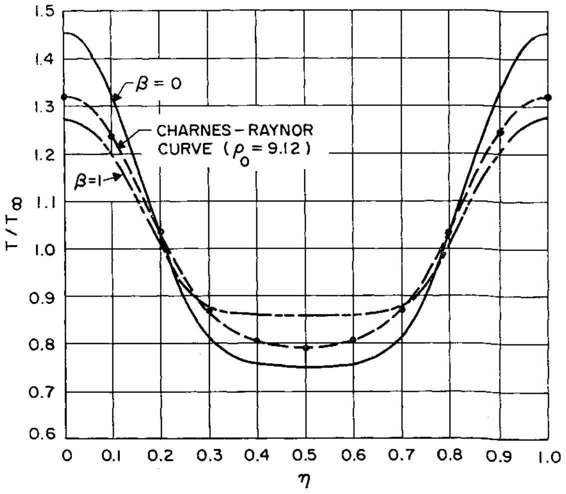
The parameter ϵ replaces the term $\partial^2T/\partial\theta^2$.

The approximation that may result from substitution of ϵ for $\partial^2T/\partial\theta^2$ in Eq. [7] may now be estimated from the behavior of the dimensionless parameter ϵ, which may be rewritten as

$$\epsilon = \frac{h}{r}\cdot\frac{k\pi T_\infty}{16r\bar{\alpha}S[1+(\pi\beta/4)]^2} = \frac{h}{r}\frac{\pi}{51}\epsilon'$$

For typical sheet metal shells with $\beta \approx 1$, ϵ' is of the order of 10^2, so that ϵ is of the order of $(h/r)\cdot 2\pi$. Then, for a sufficiently small value of h/r, $\epsilon \ll 1$, so that it may indeed be neglected in comparison with the other terms in the brackets. This will be done in the subsequent analysis, which assumption has the practical significance that the rapidly spinning, thin, spherical shells will exhibit a similarity in the thermal behavior at the equator to that of the spinning thin-walled cylinders.

When γ remains small but $f \to 0$, a comparison of temperature distribution given by the solution of Eq. [7] with that for the stationary case for $\gamma = 0$ (Eq. [9]) will be helpful. From Fig. 2 it is seen that there is a good qualitative agreement. Here, the fact that for a certain range of η the temperature is shown to remain constant is also of help.



∘∘ Solution from Eq. [A7], $\rho_0 = 9.12$

Fig. 3 Influence of internal radiation on temperature distribution; comparison with Charnes-Raynor results

Eq. [7] is similar to one that occurs in a problem that has been recently solved by Charnes and Raynor (1). For the quasi-steady state, the solution must be periodic in η:

$$\tau(0) = \tau(1)$$

and

$$\left.\frac{d\tau}{d\eta}\right|_{\eta=0} = \left.\frac{d\tau}{d\eta}\right|_{\eta=1}$$

Following Charnes and Raynor, the following definitions may be made. Let $c^{1/2} = \rho_0 = r/R$, where R = thermal radius; further, $\nu^* = a/\pi R$, the thermal velocity and $\nu_0 = 2\pi r f/\nu^*$, the dimensionless thermal velocity, so that one has

$$\frac{d^2\tau}{d\eta^2} + 2\rho_0\nu_0\frac{d\tau}{d\eta} - \rho_0{}^2\left\{\tau - \frac{3}{4} - \frac{\pi\beta}{16[1 + (\pi\beta/4)]}\right\} = -\frac{\rho_0{}^2\pi}{4}\frac{\cos^+ 2\pi\eta}{1 + (\pi\beta/4)} \quad [8]$$

Eq. [8] has the familiar form of the forced vibration equation for one degree of freedom with the damping term present but with what may be called a negative spring constant. It will be shown below that a high value of the dimensionless thermal velocity is associated with relatively slight temperature fluctuations in the spinning satellite. The solution of Eq. [8] appears in the Appendix as Eq. [A7], the results being obtained by a method suggested by Kaplan (3).

Discussion of Results

From an inspection of the solution, it is seen that as $\nu_0 \to \infty$

$$\tau \to \frac{3}{4} + \frac{1}{1 + (\pi\beta/4)}\left(\frac{\pi\beta}{16} + \frac{1}{4}\right) = 1$$

that is, $T \to T_\infty$, which could have been already anticipated from Eq. [5]. Had the term $\partial^2 T/\partial\theta^2$ been included in the form of ϵ, this would be instead $T \to T_\infty(1 - \epsilon)$. This is reasonable, because the effects of conduction in the θ direction would show themselves in reduction of temperature at the equator of the satellite.

Now, several examples will be given for the case of satellite with r = 2 ft, k = 100 Btu/hr-ft-°F, a = 3 ft²/hr, h = 0.050 in., $\sigma = 0.173 \times 10^{-8}$ Btu (hr-ft²-°F⁴)⁻¹, S = 430 Btu/ft²-hr, $\bar{\alpha} = 0.75$, and $\epsilon_i = \bar{\epsilon} = 0.9$, which corresponds roughly to a satellite with aluminum skin that is fully covered with glass-protected solar cells. The results are plotted in Fig. 2.

Consider 150 rpm as the initial rate of spin. Since here β = 1 and T_∞ = 493°R by Eq. [5], the reciprocal of the thermal radius is R^{-1} = 11.86 ft⁻¹, and the other parameters become $\rho_0 = 2 \times 11.86 = 23.72$, $\nu^* = 3 \times 11.86/3.14 = 11.32$ ft/hr, and $\nu = 1.13 \times 10^5$ ft/hr, so that $\nu_0 = 9.98 \times 10^3$.

On substituting into Eq. [A7], it is seen that τ = 1.000 to four significant figures and, therefore, $T = T_\infty$. This means that, for the rate of spin of 150 rpm, there will be practically no temperature fluctuation in the skin of the satellite.

Assume that ν_0 = 20, which corresponds to f = 0.3 rpm, all other parameters remaining the same, to determine temperature fluctuations for this borderline case between the "fast" and the "slowly" spinning satellites. Then, plotting τ vs η according to Eq. [A7], it is seen that τ = 1.023 for η = 0.820, and $T_{max} = 1.023 \cdot T_\infty$ = 504.5°R, so that the temperature fluctuation here is ±11.5°F. The example shows that even for such low spin rates the temperature fluctuations remain low for a satellite with a thin skin.

Therefore, it would be also of some interest to compare the results that were derived for the linearized spinning case with those for a stationary satellite, since, for a very low rate of spin, a comparison with the nonspinning case is physically justified. Consequently, an example will be given which is related to a practical problem.

In a magnetically oriented satellite with a 6-hr orbit,[3] the rate of spin would be $\frac{1}{3}$ rph. Assume again that all other parameters remain the same as those discussed before; therefore, ρ_0 = 23.72, but ν_0 = 0.370. By using Eq. [A7] from the Appendix, one gets $T = 1.272T_\infty$ for η = 0.95. This may be compared with the maximum temperature for a completely stationary satellite (ν_0 = 0) with a thin skin

$$T_0 = 1.217T_\infty$$

It is the exact solution for the stationary case, but with the effects of conduction disregarded, from the formula

$$T_0 = \left(\frac{S\bar{\alpha}}{4\sigma\bar{\epsilon}}\right)^{1/4}\left[\frac{\cos\theta + (\beta/4)}{1 + \beta}\right]^{1/4} \quad [9]$$

From Eq. [9] and from the corresponding curves plotted in Fig. 2, one can conclude that for the very low rates of spin the solution of Eq. [8] gives values that are about 5% too high. Since temperature fluctuations due to spinning should have been for an exact solution always less than the temperature extremes for the stationary case, this must be primarily due to effects of linearization.

Also, from the comparison of the curves given by Eq. [A7] for ν_0 = 0.370 and ν_0 = 0, it is seen that they both have the same general shape, except that the maximum of the curve for ν_0 = 0.370 is shifted in the direction of motion of the heat source.

The foregoing example shows that for a magnetically oriented satellite the temperature distribution is essentially the same as for a stationary satellite, and from the comparison with Eq. [9] it is seen that the magnitude of temperature fluctuations calculated here for that case is realistic.

Closing Statement

The results presented in this paper apply to spherical geometry and include the effects of internal radiation. The smoothing effect of internal radiation on temperature distribution can best be seen from comparison of the case for β = 1, where the effect of internal radiation is appreciable, with that for β = 0, where there is no internal radiation at all. This has been done in Fig. 3 for a nonspinning satellite.

Since the results of Charnes and Raynor apply also to the present solution with β = 0, and because in each case the final formulas are entirely different, Eq. [A7] is plotted in

[3] This would apply also to a gravitationally oriented satellite with a 3-hr orbit.

Fig. 3 for $\beta = 0$ and $\rho_0 = 9.12$, the value used in Ref. 1. It agrees with the Charnes and Raynor curve very closely. The series solution used in this paper is, therefore, for $\beta = 0$, equivalent to their own closed-form solution. For the purposes of plotting, however, the series solution proposed here offers definite advantages because of its relative simplicity.

Appendix: Solution of a Second-Order Differential Equation of Forced Vibration Type by Means of Fourier Series

Letting $x = \eta$, $y = \tau - \frac{3}{4} - (\pi\beta/16)[1 + (\pi\beta/4)]^{-1}$, Eq. [8] may be rewritten as

$$(d^2y/dx^2) + 2\rho_0\nu_0(dy/dx) - \rho_0^2 y = A_0 \cos^{+}2\pi x \qquad [A1]$$
$$0 \leq x \leq 1$$

The solution sought must be continuous and periodic in x. Regardless of its temperature history, the orbiting satellite will eventually assume a certain equilibrium that will depend only on the space environment and, possibly, on the internal power generation. In such a case the solution of Eq. [A1] will be independent of the initial conditions, and only the particular integral will have to be found.

If the right side of Eq. [A1] is changed to $A_0 \cos 2n\pi x$, the particular solution will be readily obtained by the method of undetermined coefficients:

$$y_n = -(A_0/\rho_0^2 A_n)\cos(2n\pi x + \Phi_n) \qquad [A2]$$

$$\Phi_n = \tan^{-1}\left(\frac{4\nu_0 n\pi\rho_0}{\rho_0^2 + 4n^2\pi^2}\right) \qquad [A3]$$

$$A_n = \left[\left(1 + 4n^2\frac{\pi^2}{\rho_0^2}\right)^2 + 16\frac{\nu_0^2}{\rho_0^2}n^2\pi^2\right]^{1/2} \qquad [A4]$$

On the other hand, $\cos^{+}2\pi x$ may be expanded in Fourier cosine series:

$$\cos^{+}2\pi x = \frac{1}{\pi} + \frac{1}{2}\cos 2\pi x + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{4n^2 - 1}\cos 4n\pi x \qquad [A5]$$

The expression for y_n will be the output for the corresponding input function $\cos 2n\pi x$. For the series in Eq. [A5], this will be

$$-y = \frac{A_0}{\rho_0^2}\frac{1}{\pi} + \frac{A_0}{2\rho_0^2 A_1}\cos(2\pi x + \Phi_1) + \frac{2A_0}{\pi\rho_0^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{4n^2 - 1}\frac{\cos(4n\pi x + \Phi_{2n})}{A_{2n}} \qquad [A6]$$

Using the definitions of x and y introduced before, and since

$$A_0 = -\frac{\pi}{4}\frac{\rho_0^2}{1 + (\pi\beta/4)}$$

the solution assumes the final form

$$\tau = \frac{3}{4} + \frac{1}{1 + (\pi\beta/4)}\left\{\frac{\pi\beta}{16} + \frac{1}{4} + \frac{\pi}{8A_1}\cos(2\pi\eta + \Phi_1) + \frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{4n^2 - 1}\frac{1}{A_{2n}}\cos(4n\pi\eta + \Phi_{2n})\right\} \qquad [A7]$$

The solution is rapidly convergent, since $A_n \sim n^2$, and $(4n^2 - 1)^{-1}A_{2n}^{-1} \sim n^{-4}$.

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The 1963 meeting of the Heat Transfer and Fluid Mechanics Institute (HTFMI), of which the AIAA is a sponsor, will be held June 12–14, 1963, on the campus of the California Institute of Technology, Pasadena. Papers that present recent technical and scientific advances in heat transfer, fluid mechanics, and related fields will be presented.